AP-E 430515

AD ACOLICAGO

AD

TECHNICAL REPORT ARBRL-TR-02258

BRLGRAY: THE BALLISTIC RESEARCH LABORATORY VERSION OF THE GRAY EQUATION OF STATE

Joseph F. Lacetera

August 1980





US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

Approved for public release; distribution unlimited.

DE FILE COPY

80 10 10 015

Destroy this report when it is no longer needed. Do not return it to the originator.

Secondary distribution of this report by originating or sponsoring activity is prohibited.

Additional copies of this report may be obtained from the National Technical Information Service, U.S. Department of Commerce, Springfield, Virginia 22151.

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

The use of trais names or manufacturers' names in this report does not constitute indorsement of any commercial product.

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM					
1. REPORT NUMBER 2. GOVT ACCESSION NO.		3. RECIPIENT'S CATALOG NUMBER					
TECHNICAL REFORT ARBRL-TR-	02258 AD-A091246	b .					
4. TITLE (and Subtitle)		5. TYPE OF REPORT & PERIOD COVERED					
DDI CDAY - Mar Dalldard - Dane	and Takamakama						
BRLGRAY: The Ballistic Rese Version of the Gray Equatio	•	6. PERFORMING ORG. REPORT NUMBER					
version of the dray Equation	ii or state	B. PERFORMING ONG. REPORT NUMBER					
7. AUTHOR(e)		8. CONTRACT OR GRANT NUMBER(+)					
Joseph F. Lacetera							
L		<u> </u>					
9. PERFORMING ORGANIZATION NAME AND		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT HUMBERS					
US Army Ballistic Research	Laboratory	RDT&E 1L161102AH43					
ATTN: DRDAR-BLB Aberdeen Proving Ground, MD	21005	RDIGE ILIGITUZAN43					
11. CONTROLLING OFFICE NAME AND ADDRE	21003 Fee	12. REPORT DATE					
USA Armament Research & Dev		AUGUST 1980					
Ballistic Research Laborato	•	13. NUMBER OF PAGES					
Aberdeen Proving Ground, MD	·	40					
14. MONITORING AGENCY NAME & ADDRESS	if different from Controlling Office)	15 SECURITY CLASS. (of this report)					
1		UNCLASSIFIED					
}		15a, DECLASSIFICATION/DOWNGRADING SCHEDULE					
16. DISTRIBUTION STATEMENT (of this Repor	<u> </u>	L					
IG. DISTRIBUTION STATEMENT (STATE REPORT	,						
į.							
Approved for public release	; distribution unlimit	ed.					
	Approved for public resease, distribution unlimited.						
<u></u>							
17. DISTRIBUTION STATEMENT (of the abstract	t entered in Black 20, if different fro	om Report)					
1							
}							
18. SUPPLEMENTARY NOTES							
ł							
1							
19. KEY WORDS (Continue on reverse elde if ne	eeeary and identify by block number)					
Equation of State GRA		LGRAY					
		quid-Vapor					
		over Scaling Law					
		drocode					
Tillotson Hul	1 Code						
20. ABSTRACT (Continue en reverse side if nec	essary and identify by block number)						
The BRLGRAY EOS is describ							
boundaries are shown for Al, Tillotson and HULL EOS's.	ou, and re. compariso	ous are snown with					
ILITOESON AND RULL EOS 8.							
1							
Į.							

TABLE OF CONTENTS

		Page
	LIST OF FIGURES	5
	LIST OF TABLES	7
1.	BACKGROUND	9
	A. HISTORICAL DEVELOPMENT OF GRAY EOS	9 10
2.	THE BRLGRAY EQUATION OF STATE	11
	A. Solid Region	11 14 18 19
3.	EQUATION-OF-STATE INPUT PARAMETERS	20
4.	THE DATGEN PROGRAM	22
5.	MIXED-PHASE-DATA GENERATION	23
6.	CONCLUSIONS	29
	REFERENCES	31
	APPENDIX A: LIST OF SYMBOLS	33
	APPENDIX B: LIST OF EQUATIONS	3 5 '
	DISTRIBUTION LIST	39

Acce	ssion For	
NTIS	GRA&I	
	TAB	
Unan	nounced	L) ["]
	ification_	L_J
By		
Dist:	ribution/	
	lability C	
	Avail and/	
Dist	Special	or
	pocial	1
A	1 1	1
A	İ	- 1
-	1 1	1

LIST OF FIGURES

Figur	e	Page
1.	Temperature vs Energy; Gray vs Hull	12
2.	Ambient Pressure Temperatures	12
3.	Comparison of Specific Heat Forms	15
4.	Schematic Phase Diagram for BRLGRAY	24
5.	Schematic Diagram of BRLGRAY Isotherms.	24

LIST OF TABLES

Table		Page
1.	ESTCON Parameters	21
2.	ESTCON Data	22
3.	V _{lmax} for Aluminum Isotherms Using Solid-Liquid Model	26
4.	V _{2min} for Aluminum Isotherms Using Vapor Model	26
5.	Join Volumes	27
6.	Critical Points	27
7.	Aluminum Mixed-Phase Boundary	28
8.	Copper Mixed-Phase Boundary	28
9.	Iron Mixed-Phase Boundary	29

1. BACKGROUND

A. HISTORICAL DEVELOPMENT OF GRAY EOS

The original GRAY three-phase equation of state (EOS) for metals was developed by Royce¹ at the Lawrence Livermore Laboratory(LLL). It included a Gruneisen description of the solid referenced to the solid Hugoniot², and a scaling law EOS for liquid developed by Grover³. Material in the liquid-vapor region was described by a classical hard-sphere EOS due to Young and Alder⁴ which included a van der Waals attractive term.

The solid-liquid model assumed: the entropy of the melting is independent of pressure; the temperature dependence of the specific heat in the liquid is a universal curve scaled on the melting temperature; and the pressure dependence of the melting temperature is given by a modified Lindemann law.

The Grover scaling law and the Young-Alder model were analytically joined at a volume in the range 1.3 to 1.5 times the normal volume by adding correction terms to the Young-Alder model at the low density side of the join volume.

Young⁵ subsequently introduced a modification of the original GRAY EOS to permit the calculation of the pressure in the mixed-phase liquid-vapor region; however the basic physical models and join procedure were not altered.

A substantially modified GRAY EOS, BRLGRAY, was developed in 1976 under Ballistic Research Laboratory (BRL) sponsorship by Wray⁶ at Systems, Science and Software. Modifications included a continuous model for the liquid phase, a soft-sphere model for the vapor region, and an improved join procedure for the liquid and vapor regions.

^{1.} E.B. Royce, "GRAY, A Three-Phase Equation of State for-Metals", Lawrence Livermore Laboratory, UCRL-51121, Sep 1971.

^{2.} R. N. Keeler and E. B. Royce, "Shock Waves in Condensed Media", Lawrence Livermore Laboratory, UCRL-71846, 1969.

^{3.} R. Grover, "Liquid Metal Equation of State Based on Scaling", J. Chem Phys. 55, 3435, 1971.

^{4.} D. A. Young and B. J. Alder, "Critical Point of Metals from the van der Waals Model", Phys Rev A3, 364, 1971.

^{5.} D. A. Young, "Modification of the GRAY Equation of State in the Liquid-Vapor Region" Lawrence Livermore Laboratory, UCRL-51575, Apr 15, 1974.

^{6.} William O. Wray and Robert A. Cecil, "Modified GRAY: An Improved Three-Phase Equation of State for Metals". BRL CR No. 299, April 1976. (AD #A025260)

The fluid (liquid and gas) models share a single continuous representation of the specific heat. This eliminates the artificial hot liquid region of the original model. The soft sphere representation in the vapor region yields a model which is physically realistic at higher temperatures and more compatible with the Grover model.

B. APPLICATIONS

Multi-phase EOS's are required for the numerical modelling of the response of materials to dynamic loading under which solid phase changes, melt transitions, and vapor states occur. However, equations of state currently in use for such problems include the Tillotson EOS⁷ which is used in the hydrocode HELP75⁸, and curve fits such as that used in the HULL⁹ code. These EOS's do not take into account phase changes which may be important in calculating material response to dynamic loading.

HELP75 is the most recent documented version of the HELP code. However, it does not calculate internal energy accurately and therefore cannot properly predict thermodynamic state. HULL is an Eulerian code which uses a curve fit to EOS data, but it uses the same type of energy calculation as does HELP75.

BRLHELP¹⁰ is a new improved version of HELP. It has a modification due to Schmitt¹¹ which enables it to calculate internal energies very accurately, has improved input formatting¹² and in its latest version has the BRLGRAY EOS for metals.

Janet Lacetera¹³ has done parameter studies, using the HELP code, for a variety

^{7.} J. H. Tillotson, "Metal Equations of State for Hypervelocity Impact", General Atomic Report GA-3216, July 1962.

^{8.} Laura J. Hageman, et al, "HELP, a Multi-Material Eulerian Program for Compressible Fluid and Elastic-Plastic Flows in Two Space Dimensions and Time", Systems, Science, and Software Report TR-76-45-BK2, Apr 1976.

^{9.} Richard E. Durrett and David A. Matuska, "The HULL Code, A Finite Difference Solution to the Equations of Continuum Mechanics", Air Force Armament Laboratory, AFATL-TR-78-125, Nov 1978.

^{10.} J. Lacetera, J. E. Lacetera, and J. A. Schmitt, "The BRL 7600 Version of the HELP Code", BRL REPORT ARBRL-TR-02209, Jan 80. (AD #A082804)

^{11.} J. A. Schmitt, "An Improved Internal Energy Calculation for the HELP Code and Its Implications to Conical Shaped Charge Simulations", BRL Report ARBRL-TR-02168, Jun 79. (AD #A072785)

^{12.} J. Lacetera, "BRLHELP80: NAMELIST INPUT", BRL IMR No 677, Mar 80.

^{13.} Janet Lacetera, "Study of Liner Collapse and Jet Formation for Various Hemispherical Shaped-Charge Designs", BRL Draft Report.

of problems. Agreement with experiment for these calculations has been quite good and indicates that the HELP code provides sufficiently accurate information to be used as a design optimization tool in hydrocode applications. However, the EOS imbedded in the code does not adequately represent the effects of phase changes. For applications where such phenomena are important a more sophisticated EOS is appropriate.

We have made comparisons between BRLGRAY, Tillotson, and the HULL EOS's. In the solid state the agreement between these EOS's and with experiment is excellent. Significantly, BRLGRAY uses no more computer time than do the others for solid state calculations and outperforms the others in the melt transition and at higher temperatures. A comparison of the BRLGRAY EOS and that used in the HULL code, for temperature as a function of energy for copper, is shown in Figure 1. The phase transition, in BRLGRAY, begins at an energy of 4.62 ergs/g and is complete at an energy of 6.67 ergs/g. This heat of transition of 2.05 ergs/g is very close to the values given in the various handbooks. Figure 2 is another illustration of the BRLGRAY melt-phase transition: ambient-pressure temperature profiles are shown for the copper and aluminum. The melt temperatures are 1356 K for copper and 932 K for aluminum. The melt transition is simply not calculated by the other EOS's.

2. The BRLGRAY Equation of State*

A. Solid Region

This portion of the EOS remains unmodified. Grover's scaling law includes a Gruneisen description of the solid referenced to the experimental Hugoniot. The volume dependence of gamma in the solid is linear. Energy and pressure in this region are given by

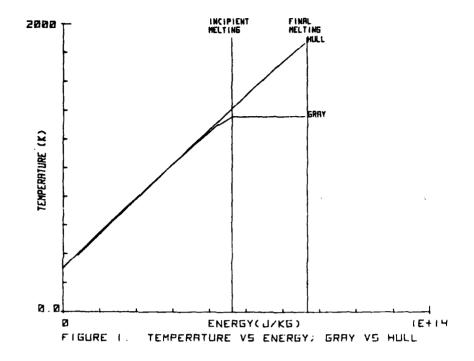
$$E_s(T,V) = E_o(V) + 3R' + \frac{G'T^2}{2},$$
 (1)

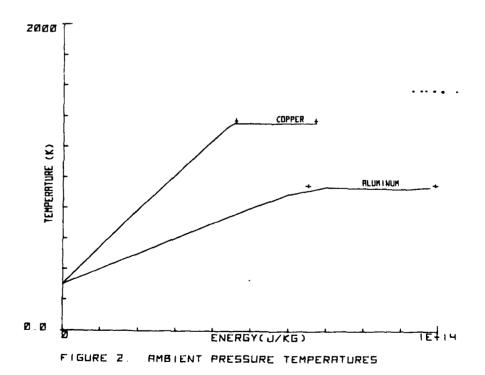
and

$$P_s(T,V) = P_o(V) + 3\gamma_s(V) \frac{R'T}{V} + \frac{1}{2}\gamma_e \frac{G'T^2}{V},$$
 (2)

where R' and G' are the atomic-weight scaled gas constant and electronic energy coefficient respectively, $P_0(V)$ and $E_0(V)$ are reference values at 0° K, and $\gamma_s(V)$ is the volume-dependent Gruneisen coefficient for the solid.

^{*} A list of symbols for the EOS is given in Appendix A, and a complete list of the EOS equations is given in Appendix B.





The atomic-weight scaled electronic energy coefficient is given by

$$G' = \frac{\pi^2 N_e k R'}{2E_f},$$
 (3)

where the Fermi energy, Er is given by

$$E_f = 26 \left[\frac{N_e}{WV_o} \right]^{2/3}$$
, (4)

where W is atomic weight, and Ne is number of free electrons.

The complete solid EOS is described by Royce¹. The experimental Hugoniot is assumed to have the standard form:

$$U_{s} = C + SU_{n}, \tag{5}$$

$$P_{H}(V) = \frac{\rho_{o}C^{2}x}{(1-Sx)^{2}},$$
 (6)

and

$$E_{H}(V) = \frac{1}{2}P_{H}(V)[V_{o}-V] + E_{oH},$$
 (7)

where x is a compression variable, and C and S are Hugoniot parameters. To reference the EOS to the Hugoniot, $\Gamma(T,V)$ is defined by the Gruneisen-like equation

$$P(T,V) = P_o(V) + \frac{\Gamma(T,V)}{V} [E(T,V) - E_o(V)],$$
 (8)

and the Hugoniot in the solid is written

$$P_{H} = P_{o}(V) + \frac{\gamma_{s}(V)}{V} [E_{H}(V) - E_{o}(V)],$$
 (9)

where γ_s is the volume-dependent Gruneisen parameter in the solid.

Equations 7-9 are combined, using $\Gamma(T=0,V)=\gamma_s(V)$ to give

$$P(T,V) = P_{H}(V)[1 - \frac{\gamma_{s}(V)}{2V}(V_{o} - V)] + \frac{\gamma_{s}(V)}{V} [E(T,V) - E_{oH}] + \left[\frac{\Gamma(T,V) - \gamma_{s}(V)}{V} \right] [E(T,V) - E_{o}(V)]$$
(10)

where E_{oH} is the energy at the foot of the Hugoniot.

P(T.V) is then written as the sum of two parts

$$P(T,V) = P_G(V,E) + P_C(T,V),$$
 (11)

where

$$P_{G}(V,E) = P_{H}(V) \left[1 - \frac{\gamma_{s}(V)}{2V} (V_{o} - V) \right] + \frac{\gamma_{s}(V)}{V} \left[E(T,V) - E_{oH} \right]$$
 (12)

is a Gruneisen-like equation of state for the solid, and

$$P_{C}(T,V) = \left[\frac{\Gamma(T,V) - \gamma_{s}(V)}{V}\right] \left[E(T,V) - E_{p}(V)\right]$$
 (13)

is a correction term.

Since the EOS is referenced to the experimental Hugoniot, an explicit expression for Po(V) is not needed. However, the calculation of the temperature, as a function of energy, which is needed for the calculation of P_c requires a knowledge of $E_o(V)$. This is given by $Royce^1$ to be

$$E_{o} = \frac{C^{2} - x^{2}}{2(1 - S_{x})} \left[S + \frac{S}{3}x + \frac{S^{2}}{6} (1 - \frac{\gamma_{o}}{S})x^{2} \right] + E_{oo}(1 + \gamma_{o}x) + E_{oH}$$
 (14)

which he estimates to be correct to +10 percent for compressions up to 2.

B. Melt and Liquid Regions

The Grover liquid-metal EOS is based on the observation that the entropy of melting can be scaled on the atomic weight using a single constant (2.32 cal/g-atom deg) for all metals and that the liquid specific heat has a universal dependence on the scaled temperature (T/T_m) . These observations are supported by experimental data. Finally, the Lindemann law is used to establish the volume dependence of the melting temperature.

In BRLGRAY the fluid specific heat has a universal hyperbolic dependence on the scaled temperature:

$$C_V = \frac{3R}{2} \left\{ 1 + \left[\alpha \frac{T}{T_m} + 1 \right]^{-1} \right\}, \quad \alpha = 0.1, \ T > T_m$$
 (15)

This is illustrated in Figure 3, along with the Grover original linear specific heat for the liquid. The Wray form limits asymptotically to the ideal gas value.

The entropy in the liquid region is obtained from the thermodynamic relation

$$\frac{\partial S}{\partial T}\Big|_{V} = \frac{C_{V}}{T},$$

Using Equation 5 and integrating Equation 6 with respect to temperature gives, following Wray's notation,

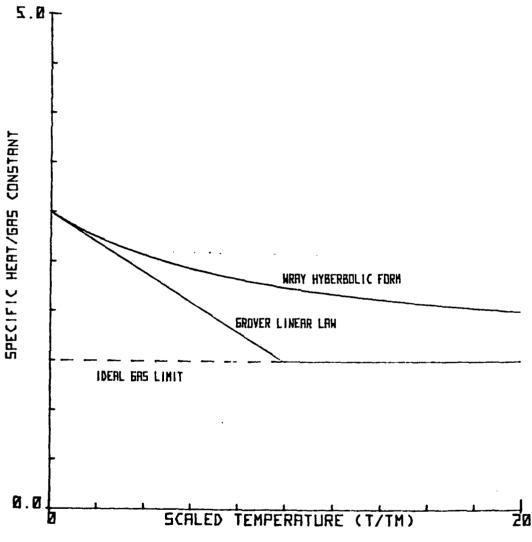


FIGURE 3. COMPARISON OF SPECIFIC HEAT FORMS

$$S_1 = 3RinT - \frac{3R}{2}ln\left[\alpha \frac{T}{T_m} + 1\right] + f(V)$$

Similarly for the solid,

$$S_e = 3R \ln T + h(V),$$

where f and h are undetermined functions of volume. The entropy of melting is then defined as

$$\Delta S = f(V) - h(V),$$

where ΔS is determined from experimental data².

The entropy in the liquid is then

$$S_i = S_s - \frac{3R}{2} ln \left[\alpha \frac{T}{T_m} + 1 \right] + \Delta S.$$

In the two-phase melt region,

$$S_m = \nu S_1 + (1 - \nu) S_S$$

where

$$\nu = \frac{T - T_s(V)}{T_1(V) - T_s(V)}$$
, and $T_m = \frac{T_s + T_1}{2}$

Since, in the melt region, $T \approx Tm$, the melt entropy is approximated by

$$S_{\rm m} - S_{\rm s} + \nu \left(\Delta S - .143R \right).$$
 (16)

The Helmholtz free energy A is obtained from the thermodynamic relation

$$S = -\left[\frac{\partial A}{\partial T}\right]_{V},$$

which may be integrated to give

$$A = E_o(V) - \int SdT,$$

where Eo(V) is the cold compression energy. This yields, after substitution and integration,

$$A_m = A_s = \nu^2 (\Delta T/2)(\Delta S - .143R),$$
 (17)

where $\Delta T = T_i(V) - T_s(V)$ and from A = E - TS it follows that

$$E_{\rm m} = E_{\rm s} + \nu [T - \nu (\Delta T/2)](\Delta S - .143R),$$
 (18)

where $E_s = A_s + TS_s$ is the extrapolated energy in the solid.

The pressure in the melt region is obtained by applying the relation

$$P = -\left[\frac{\partial A}{\partial V}\right]_{T}$$

to Equation 17, and using the approximations

$$\frac{d(\Delta T)}{dV} = 0, \quad \frac{dT_s}{dV} \approx \frac{dT_m}{dV}$$

along with the volume dependence of the melt temperature, given by the Lindemann law,

$$\lambda = \frac{-d(\ln T_m)}{d\ln V}$$

to get

$$P_{\rm m} = P_{\rm s} + \nu \frac{\lambda T_{\rm m}}{V} (\Delta S - .143R). \tag{19}$$

The same procedure, starting with the entropy in the liquid region yields similar equations for the liquid region. These are:

$$S_{l} = S_{s} - \frac{3R}{2} ln \left[\frac{\alpha T}{T_{m}} + 1 \right] + \Delta S;$$
 (20)

$$A_{1} = A_{S} - \frac{\Delta T}{2} (\Delta S - .143R) - (T - T_{m}) \left[\Delta S + \frac{3R}{2} \right]$$

$$+ \frac{3RT_{m}}{2\alpha} \left\{ \left[\frac{\alpha T}{T_{m}} + 1 \right] \ln \left[\frac{\alpha T}{T_{m}} + 1 \right] - (\alpha + 1) \ln(\alpha + 1) \right\}; \qquad (21)$$

$$E_{l} = E_{s} + T_{m}\Delta S + .143R\frac{\Delta T}{2} + \frac{3RT_{m}}{2\alpha} \left\{ ln \left[\alpha \frac{T}{T_{m}} + 1 \right] - (\alpha + 1)ln(\alpha + 1) - \left[\alpha \frac{T}{T_{m}} - 1 \right] \right\}; \text{and}$$
 (22)

$$P_{l} = P_{s} - \frac{\lambda T_{m}}{V} \Delta S$$

$$- \frac{3R\lambda}{2\alpha} \frac{T_{m}}{V} \left\{ \ln \left[\alpha \frac{T}{T_{m}} + 1 \right] - (\alpha + 1) \ln(\alpha + 1) - \left[\alpha \frac{T}{T_{m}} - 1 \right] \right\}. \tag{23}$$

All of these equations are listed for reference in Appendix B.

C. The Vapor Region

The classical van der Waals equation of state has the form

$$P_{v} = \frac{RT}{V - V_{b}} - \frac{a}{V^{2}},$$

where a is the coefficient of attractive potential, and V_b is the hard sphere excluded volume which represents the repulsive potential by increasing the pressure at small volumes. The Young-Alder modified form of the van der Waals EOS has the form

$$P_{v} = \frac{RT}{V} \left[\frac{1 + \eta + \eta^{2} - \eta^{3}}{(1 - \eta)^{3}} \right] - \frac{a}{V^{2}},$$
 (24)

where $\eta = \frac{V_b}{V}$.

The first term in Equation 24 is an algebraic approximation 14 to the hard sphere pressure, which was developed from an analysis 15 of the reduced virial series, and has been shown to describe rigid-sphere behaviour better than any other existing form.

An ideal gas caloric equation of state:

$$E_{v} = \frac{3RT}{2} - \frac{a}{V}$$

completed the model.

Wray⁶ noted that the Gruneisen coefficient,

$$G = \frac{2}{3} \left[\frac{1 + \eta + \eta^2 - \eta^3}{(1 - \eta)^3} \right]$$

in the Young-Alder model yielded an unreasonably high value at the join volume between the Grover and Young-Alder models, and being independent of temperature did not limit properly as temperature approached infinity. His soft-sphere model allows $V_{\rm b}$ to vary as a function of temperature, such that

$$V_b = V_b(T),$$
 $\lim_{T \to \infty} V_b = 0,$ $\lim_{T \to \infty} \frac{dV_b}{dT} = 0$

thus yielding the correct ideal gas, high temperature limit of G=2/3.

^{14.} N. F. Carnahan and K. E. Starling, Journal of Chemical Physics, 51, 635, 1969.

^{15.} B. J. Alder and T. E. Wainwright, Journal of Chemical Physics, 33, 1439, 1960.

The generalized Rowlinson method¹⁶ was used to relate the temperature dependence of the sphere diameter to realistic fluid behaviour, so the soft sphere diameter is calculated according to

$$d = \int_0^\infty \left[1 - e^{\frac{-u(r)}{kT}}\right] dr,$$

where u(r) is the repulsive potential. Since Grover showed his liquid scaling law to be in close agreement with computer experiments in the liquid phase for particles having an inverse-twelve potential, the following potential is used,

$$u(r) = \epsilon \left(\frac{\sigma}{r}\right)^{12}$$

where ϵ is the Fermi energy, and σ is the hard-sphere diameter. This yields, eventually,

$$d = 1.0556 \left[\frac{\epsilon \sigma^{12}}{kT} \right]^{1/12}$$

for the soft-sphere diameter.

Using the appropriate thermodynamic relations, the thermal energy E and the Helmholtz free energy A can be derived as

$$E_{v} = \frac{3RT}{2} - \frac{2RT^{2}}{V} \frac{(2-\eta)}{(1-\eta)^{3}} \frac{dV_{b}}{dT} - \frac{a}{V} + E_{s}$$
 (25)

and

$$A_{v} = \left[\frac{7 - 6\eta + \eta^{2}}{2(1 - \eta)^{3}} - \ln V \right] RT - \frac{3RT}{2} \ln T - \frac{a}{V} + E_{s}$$
 (26)

The complete vapor EOS for BRLGRAY is listed in Appendix B.

D. Join Region

In Wray's join region, between the liquid and vapor states, a trigonometric mixing function was applied to the Helmholtz free energy calculated in each of the two states. All other functions of interest were then obtained through the application of the

Market Committee of the
^{16.} H. L. Anderson, J. D. Weeks and D. Chandler, "Relationship between the Hard-Sphere Fluid and Fluids with Realistic Repulsive Forces", Physical Review, Vol. 4, No. 4, October 1971.

appropriate thermodynamic relations. However, this method is replaced in BRLGRAY with a simple volume weighted interpolation scheme which gives better join region pressures by avoiding less accurately calculated functions in the join region.

3. EQUATION OF STATE INPUT PARAMETERS

The EOS input parameters are stored in a two-dimensional array, ESTCON(I,N), where I is the parameter index and N is the material index. This array is defined in Table 1. Table 2 lists the ESTCON data for the metals Al, Cu, and Fe. It should be noted that V_1 and V_2 in the ESTCON array (I=14 and I=15) are defined in the BRLGRAY EOS and represent the volume limits of the new join region. They replace the single join volume, $V_1(I=25)$, of the original GRAY EOS.

The ESTCON array is maintained in data statements in a subroutine named ESTKON. Note that λ_e has the constant value .6667 for all metals and ΔS has the constant value 9.637E-05 for all metals.

·	TABLE 1					
	ESTCON PARAMETERS					
Index	DCIIIIIIII					
1	$ ho_{ m o}$	Normal density Kg/m ³				
2-5		Unused				
6	C_{o}	Bulk Sound Speed (cm/s)				
7	AMU	Shear Modulus (d/cm²)				
8	$\mathbf{Y_o}$	Yield Strength (d/cm ²)				
9	σ	Spall Strength (d/cm ²)				
10	$\mathbf{E_s}$	Sublimation Energy (erg/g)				
11	\mathtt{E}_{LV}	Incipient Vaporization Energy (erg/g)				
12	E_LM	Complete Melt Energy (erg/g)				
13	E_SM	Incipient Melt Energy (erg/g)				
14	$\overline{\mathbf{v_1}}$	Liquid-Join Boundary (m ³ /Kg)				
15	V_2	Join-Vapor Boundary (m ³ /Kg)				
16	s	Hugoniot Parameter: (cm/µsec)				
17	λ_{o}	Lattice gamma: $\lambda(V) = \lambda_0 - ax$				
18	a	Defined in ESTCON(17,N)				
19	$\lambda_{\mathbf{e}}$	Electronic gamma				
20	g _e	Electronic energy coefficient				
	·	(mbar cm ³ /mole deg ²)				
21	T_{MO}	Melt temperature (°K) at V=V				
22	E _{oH}	Energy at V=V _o ,T=300°K,				
	0.1	P=0, (mbar cm ² /g)				
23	E_{∞}	Energy difference between EoH and				
		energy at V=V ₀ ,T=0(mbar cm ³ /g)				
		Default value: E _{co} =				
		$-300(3*8.134*10^{-5}+150g_{e})/W$				
24	W	Atomic weight (g/mole)				
25		Unused				
26	$\mathbf{v}_{\mathtt{b}}$	Vapor exclusion volume (m³/Kg)				
27	ay	Coefficient of attractive potential				
	•	mbar(cm ³ /mole) ²				
28	$T_{\mathbf{c}}$	Critical temperature (°K)				
29	ΔŠ	Entropy of melting mbarcm ³ /mole ⁰)				
30	EOSFLG	Equation of state flag				

	TA	BLE 2	*
	ESTC	ON DATA	
Symbol	Al	Cu	Fe
$ ho_{ m o}$	2.7	8.94	7.87
C_{o}	5.44E+05	3.94E + 05	5.13E+05
E_{s}	1.18E+11	5.37E+10	6.80E+10
ELV	2.65E+10	4.79E+10	*******
E_{LM}	1.06E+10	6.67E+09	1.49E+10
E _{SM}	6.62E + 09	4.62E+09	1.16E+10
$\mathbf{v_i}$.40	.12	.14
V_2	.58	.16	.18
S	1.34	1.49	1.40
λ_{o}	2.18	1.97	1.80
a	1.70	1.50	1.50
λ_e	.6667	.6667	.6667
g_e	8.70E-09	4.90E-09	9.60E- 09
T_{M0}	1220	1790	2385
W	26.98	63.54	55.85
V_b	.190	.056	.066
$\mathbf{a_Y}$	47.0	35.0	31.0
T_c	10633	11407	9753

4. THE DATGEN PROGRAM

In order to use the BRLGRAY EOS in the liquid-vapor, mixed-phase region it is necessary to calculate the four critical constants:

$$V_c$$
, T_c , and $P_c(V_c, T_c)$ and $E_c(V_c, T_c)$,

and the mixed-phase boundary. We calculate these using a program which is based on the main subroutine given by Young⁵.

Initially the critical volume is estimated to be 4.35 times the boiling point volume. An iteration process, using pressures P(V,T), generated by the EOS, then locates the critical isotherm and the volume for which $\Delta P/\Delta V$ is zero on the critical isotherm, the critical volume. At this point $P=P_c$ and E_c is calculated by the EOS. The code then chooses a set of equally spaced temperatures between T_c and T_{mo} , the melting

temperature parameter, to draw tie lines connecting the high density liquid with the low density vapor. The corresponding pressures and equilibrium liquid and vapor densities are computed by Maxwell construction. The locus of these tie-line end points then forms the dome-shaped, mixed-phase boundary in PV space. The corresponding energies $E(V_{min},T)$ and $E(V_{max},T)$ are also computed for each temperature and the data is stored in a permanent file for access by the EOS. The associated variable names are:

TMP:temperature on a tie line

VMN:volume on liquid end of tie line

VMX:volume on vapor end of tie line

EVMN:energy at VMN and TMP

EVMX:energy at VMX and TMP

PRES:pressure on a tie line

(°K)

(m³/Mg)

(m³/Mg)

(erg/g)

(erg/g)

(erg/g)

where d/cm² are working units in the code, and are equivalent to 10⁻¹ Pascals.

The DATGEN program also has an isotherm driver subroutine which generates isotherms in P-V space using the EOS along with subroutine ET⁵. Subroutine ET calculates the specific internal energy consistent with the driver values of specific volume and temperature. The BRLGRAY EOS is then called to calculate the pressure. This program is used to determine the volume limits of the new join region in BRLGRAY. Wray⁶ performed such calculations for three metals: Al, Fe, and U. We have made calculations for seventeen materials using slightly different procedures, and in this report have presented data for Al, Fe, and Cu. These calculations are described in the next section of this report.

5. MIXED-PHASE DATA GENERATION

Figure 4 shows a P-V plot of the regions defined in BRLGRAY. Pressure and temperature from the liquid and vapor phases are volume weighted in the join region.

Figure 5 shows a schematic diagram of representative isotherms in the join region. The values of V_1 and V_2 are chosen such that pressure maxima and minima do not occur along an isotherm in the join region. Such constructs would make it impossible for the DATGEN program to find the critical isotherm in subsequent mixed-phase boundary calculations.

Wray used a trial and error procedure in which isotherms were generated for various values of V_1 and V_2 . We used a similar isotherm driver but ran separate cases in which the EOS was constrained to either the solid-liquid or the vapor model. Analysis of these pressure isotherms then allowed a good initial estimate of V_1 and V_2 .

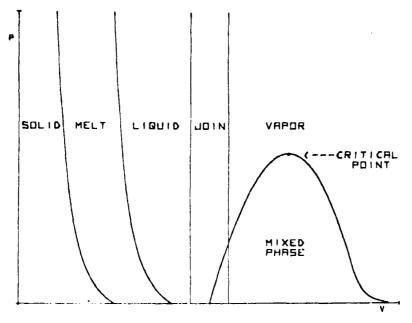


FIGURE 4 SCHEMATIC PHASE DIAGRAM FOR BREGRAY

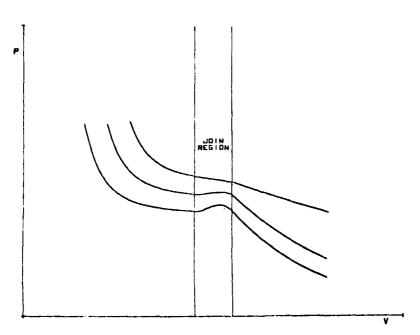


FIGURE S. SCHEMATIC DIAGRAM OF BREGRAY ISOTHERMS

The isotherms generated by the solid-liquid model were well behaved up to a volume at which the model failed catastrophically. These isotherms gave a good indication of the maximum value which V_1 could take. Comparison of these isotherms with isotherms generated by the vapor model then allowed an estimate of the best value of V_2 .

Isotherms generated using these values of V_1 and V_2 generally gave positive pressures in the join region, without maxima or minima, for the temperatures of interest.

Alternatively, once these parameters have been generated for a number of metals one may estimate V_1 and V_2 for a new metal using the following scaling rule:

$$\rho_0 V_1 = 1.1 = C_1$$

$$\rho_0 V_2 = 1.4 = C_2$$

where the C_i are average values of $\rho_0 V_i$. The V_i estimated in this way will need very little adjustment after observation of the isotherms in the join region using the complete EOS.

The driver routine was programmed to generate isotherms which ranged from 20,000°K down to 2,000°K in decrements of 2,000°K, and to lower temperatures in smaller steps. Volumes ranged from .01 m³/Mg to beyond the mixed-phase region in equal increments of ln V.

Aluminum isotherms were generated starting with a specific volume of .01 and incremented until the calculation failed. Inspection of the code output then determined maximum values of V_1 for each isotherm. These values, shown in Table 3, ranged from .67 m³/Mg at 20,000°K to .37 m³/Mg at 300°K. In anticipation of being able to extend the validity of the model to below 1,000°K, at these volumes, we chose V_1 to be 40 m³/Mg.

Vapor-model aluminum isotherms were then generated starting with specific volumes equal to .355 m³/Mg and extending beyond the expected value of V_2 . Comparison of these isotherms with those generated with the solid-liquid model showed a crossover point for each isotherm which was taken as the minimum value of V_2 in each case. These values, shown in Table 4, ranged from .58 m³/Mg at 20,000°K to .54 m³/Mg at 6,000°K. Thus .58 m³/Mg was interpreted as an upper bound on V_2 , ensuring that crossover would occur, within the join region, for all temperatures below 20,000°K.

Finally, a set of isotherms were generated in the code's normal mode (in which it determines the thermodynamic phase and chooses the corresponding model of the EOS) using the values V_1 =.40 m³/Mg and V_2 =.58 m³/Mg. These isotherms gave positive pressures in the join region without maxima or minima for temperatures down to 1,000°K. A set of isotherms using V_1 = .41 m³/Mg showed a minimum in the join region at 4,000°K, and a set using .37 m³/Mg showing minima in the 1,000°K isotherm.

TABLE 3 V _{lmax} for Aluminum Isotherms Using Solid-Liquid Model			
Temperature (K)	V _{1max} (m³/Mg)	Phase	
20,000	.67		
15,000	.66	Liquid	
10,000	.62	·	
8,000	.61	Melt	
6,000	.58		
4,000	.54		
2,000	.45	Solid/Melt/Liquid	
1,000	.41		
500	.37		
300	.37		

	TABLE 4	
V ₂ min F	or Aluminum Isotherms Using	Vapor Model
Temperature (K)	V _{2min} (m³/Mg)	Phase
20,000 10,000	.58 .56	Vapor
8,000 6,000	.55 .55 .54	· upo :

 V_1 and V_2 were determined, using the same approach, for the other metals. These are shown in Table 5 along with the values for Al.

Once the volume limits of the join region were determined for each of the metals, the DATGEN program was used to calculate the critical point, and subsequently the mixed-phase boundary. Critical point data for the five metals is shown in Table 6.

Finally the DATGEN program is used to generate the mixed-phase boundaries.

This data is shown in Tables 7 - 9.

TABLE 5 JOIN VOLUMES			
Metal V ₁			
Al	.40	.58	
Cu	.12	.16	
Fe	.14	.18	

		TABLE 6		
		CRITICAL POI	NTS	
Metal	Vc (m ³ /Mg)	Tc (°K)	Pc (10 ⁸ *P)	Ec (.1mJ/g)
Al	1.460	7180	5.34E-03	1.25E+09
Cu	.413	8880	9.88E-03	6.34E + 10
Fe	.498	9910	1.04E-02	7.24E + 10

TABLE 7						
	ALUMINUM MIXED-PHASE BOUNDARY					
T_{c}	V_c	P _c	E_{c}			
7178	1. 46	5.34E+09	1.25E+11			
TMP	VMN	VMX	EVMN	EVMX	PRES	
6.64E+03	8.93E-01	3.03E+00	1.07E+11	1.35E+11	3.83E+11	
6.09E+03	7.63E-01	4.76E+00	9.87E+10	1.37E+11	3.83E+09	
5.55E+03	6.85E-01	7.69E+00	9.16E+10	1.38E+11	1.68E+09	
5.01E+03	6.33E-01	1.25E + 01	8.55E+10	1.37E+11	9.98E+08	
4.47E+03	5.95E-01	2.00E+01	8.01E+10	1.36E+11	5.31E+08	
3.93E+03	5.68E-01	3.33E+01	7.50E + 10	1.35E+11	2.40E+08	
3.39E+03	5.56E-01	1.11E+02	7.01E+10	1.33E+11	8.86E+07	
2.85E+03	5.46E-01	2.50E+02	6.52E + 10	1.31E+11	3.21E+07	
2.30E+03	5.38E-01	1.11E+03	5.81E+10	1.28E+11	5.70E+06	
1.76E+03	5.10E-01	1.43E+04	4.14E+10	1.26E + 11	3.64E+05	
1.22E+03	4.22E-01	3.33E+06	1.40E+10	1.24E+11	9.88E+02	

		TAF	BLE 8				
COPPER MIXED-PHASE BOUNDARY							
T _c	$\mathbf{v}_{\mathbf{c}}$	P_c	$\mathbf{E}_{\boldsymbol{c}}$		j		
8492	.435	8.91E+09	6.33E+11				
TMP	VMN	VMX	EVMN	EVMX	PRES		
7.83E+03	2.69E-01	8.93E-01	5.49E + 10	6.85E + 10	6.50E+09		
7.22E+03	2.31E-01	1.39E+00	5.08E+10	6.96E+10	4.55E+09		
6.62E+03	2.09E-01	2.13E + 00	4.74E + 10	6.99E + 10	3.02E+09		
6.01E+03	1.93E-01	3.33E+00	4.45E+10	6.98E + 10	1.87E+09		
5.41E+03	1.81E-01	5.55E+00	4.18E+10	6.93E + 10	1.05E+09		
4.81E+03	1.72E-01	1.00E+01	3.94E+10	6.86E + 10	5.23E+08		
4.20E+03	1.65E-01	2.00E+01	3.73E + 10	6.78E + 10	2.17E+08		
3.60E+03	1.59E-01	5.00E+01	3.52E+10	6.68E + 10	6.93E+07		
2.98E+03	1.55E-01	2.00E+02	2.94E + 10	6.57E+10	1.74E+07		
2.39E+03	1.49E-01	1.67E+03	2.28E+10	6.46E + 10	1.83E+06		
1.79E+03	1.38E-01	5.00E+04	1.33E+10	6.34E+10	3.50E+04		

		TAI	BLE 9					
IRON MIXED-PHASE BOUNDARY								
T _c	V_c	P_c	${\tt E_c}$					
9913	.498	1.04E+10	7.23E+10					
TMP	VMN	VMX	EVMN	EVMX	PRES			
9.23E+03	3.12E-01	1.01E+00	6.15E + 10	7.94E + 10	7.69E+09			
8.54E+03	2.67E-01	1.54E + 00	5.60E+10	8.08E + 10	5.46E+09			
7.86E+03	2.41E-01	2.33E + 00	5.16E+10	8.13E+10	3.70E+09			
7.18E+03	2.23E-01	3.57E+00	4.78E + 10	8.11E+10	2.35E+09			
6.49E+03	2.09E-01	5.88E+00	4.43E + 10	8.06E + 10	1.38E+09			
5.81E+03	1.99E-01	1.00E + 01	4.11E + 10	7.98E + 10	7.20E+08			
5.12E+03	1.90E-01	2.00E+01	3.82E + 10	7.87E + 10	3.24E+08			
4.44E+03	1.83E-01	5.00E+01	3.36E+10	7.75E + 10	1.14E+08			
3.75E+03	1.73E-01	1.67E+02	2.63E+10	7.61E+10	3.14E+07			
3.07E+03	1.49E-01	1.11E+03	1.76E+10	7.47E + 10	3.83E+06			
2.39E+03	1.43E-01	2.00E+04	1.36E+10	7.32E+10	1.43E+05			

6. Conclusion

Equation-of-state input parameters have been generated for seventeen different materials. Data has been shown here for Al, Cu, and Fe. The EOS has been installed in the BRLHELP code and comparisons have been made with the Tillotson EOS and the temperature fit in the HULL code. In the solid state the agreement between BRLGRAY, Tillotson, the HULL EOS's, and with experimental data is excellent. Significantly, BRLGRAY uses no more computer time for the solid state than do the other EOS's, while outperforming them in calculating phase change phenomena.

The BRLGRAY EOS is being made available for use by hydrocode users at the BRL and at the ARC computers at Huntsville, as part of the BRLEOS code system¹⁷.

^{17.} Joseph Lacetera and W. Buchheister, "BRLPUFF and the BRLEOS Code System" BRL IMR No. 655, Aug 1979.

REFERENCES

- 1. E.B. Royce, "GRAY, A Three-Phase Equation of State for Metals", Lawrence Livermore Laboratory, UCRL-51121, Sep 1971.
- 2. R. N. Keeler and E. B. Royce, 'Shock Waves in Condensed Media" Lawrence Livermore Laboratory, UCRL-71846, 1969.
- 3. R. Grover, "Liquid Metal Equation of State Based on Scaling", J. Chem Phys, 55, 3435, 1971.
- D. A. Young and B. J. Alder, "Critical Point of Metals from the van der Waals Model", Phys Rev A3, 364, 1971.
- 5. D. A. Young, "Modification of the GRAY Equation of State in the Liquid-Vapor Region", Lawrence Livermore Laboratory, UCRL-51575, Apr 15, 1974.
- 6. William O. Wray and Robert A. Cecil, "Modified GRAY: An Improved Three-Phase Equation of State for Metals". BRL CR No. 299, April 1976. (AD #A025260)
- 7. J. H. Tillotson, "Metal Equations of State for Hypervelocity Impact", General Atomic Report GA-3216, July 1962.
- 8. Laura J. Hageman, et al., "HELP, A Multi-Material Eulerian Program for Compressible Fluid and Elastic-Plastic FLows in Two Space Dimensions and Time", Systems, Science and Software Report TR-76-45-BK2, April 1976.
- Richard E. Durrett and David A. Matuska, "The HULL Code, A Finite Difference Solution to the Equations of Continuum Mechanics", Air Force Armament Laboratory, AFATL-TR-78-125, November, 1978.
- J. Lacetera, J. E. Lacetera, and J. A. Schmitt, "The BRL 7600 Version of the HELP Code", BRL REPORT ARBRL-TR-02209, Jan 80. (AD #A082804)
 J. A. Schmitt, "An Improved Internal Energy Calculation for the HELP Code and Its Implications to Conical Shaped Charge Simulations", BRL Report ARBRL-TR-02168, Jun 79. (AD #A072785)
- 12. J. Lacetera, "BRLHELP80: NAMELIST INPUT", BRL IMR No 677, Mar 80.
- 13. Janet Lacetera, "Study of Liner Collapse and Jet Formation for Various Hemispherical Shaped-Charge Designs", BRL Draft Report.
- 14. N. F. Carnahan and K. E. Starling, Journal of Chemical Physics, 51, 635, 1969.

- 15. B. J. Alder and T. E. Wainwright, Journal of Chemical Physics, 33, 1439, 1960.
- H. L. Anderson, J. D. Weeks and D. Chandler, "Relationship between the Hard-Sphere Fluid and Fluids with Realistic Repulsive Forces", Physical Review, Vol. 4, No. 4, October 1971.
- 17. Joseph Lacetera and W. Buchheister, "BRLPUFF and the BRLEOS Code System" BRL IMR No. 655, Aug 1979.

APPENDIX A

All symbols used in this report are defined where they first appear in the text. However, selected symbols are listed here for convenience.

(A-3)
$$\gamma_s(V)$$
 volume dependent Gruneisen coefficient for solid

(A-4)
$$\gamma_e$$
 electronic gamma (normally 2/3)

$$(A-6)$$
 N_e number of free electrons

(A-11)
$$x = \frac{V_o - V}{V}$$
 compression variable

(A-12)
$$E_{OH}$$
 Energy at $V=V_0$, $T=300^{\circ}K$, $P=0$

(A-13)
$$E_{\infty}$$
 Energy difference between E_{oH} and energy at $V=V_o$, $T=0$ Default = $-300(3*8.134*10^{-5}+150g_e)/W$

$$(A-14)$$
 α constant in hyperbolic form of specific heat

APPENDIX B

$$E_s(T,V) = E_o(V) + 3R' + \frac{G'T^2}{2}$$
 (1)

$$P_s(T,V) = P_o(V) + 3\gamma_s(V) \frac{R'T}{V} + \frac{1}{2}\gamma_e \frac{G'T^2}{V}$$
 (2)

$$G' = \frac{\pi^2 N_e kR'}{2E_r} \tag{3}$$

$$E_{\rm f} = 26 \left[\frac{N_{\rm e}}{WV_{\rm o}} \right]^{2/3} \tag{4}$$

$$U_{s} = C + SU_{p} \tag{5}$$

$$P_{H}(V) = \frac{\rho_{o}C^{2}x}{(1-Sx)^{2}}$$
 (6)

$$E_{H}(V) = \frac{1}{2}P_{H}(V)[V_{o}-V] + E_{oH}$$
 (7)

$$P(T,V) = P_o(V) + \frac{\Gamma(T,V)}{V} [E(T,V) - E_o(V)]$$
 (8)

$$P_{H} = P_{o}(V) + \frac{\gamma_{s}(V)}{V} [E_{H}(V) - E_{o}(V)]$$
 (9)

$$P(T,V) = P_H(V)[1 - \frac{\gamma_s(V)}{2V}(V_o - V)] + \frac{\gamma_s(V)}{V}[E(T,V) - E_{oH}]$$

$$+ \left[\frac{\Gamma(T,V) - \gamma_s(V)}{V} \right] \left[E(T,V) - E_o(V) \right]$$
 (10)

$$P(T,V) = P_G(V,E) + P_C(T,V)$$
 (11)

$$P_{G}(V,E) = P_{H}(V) \left[1 - \frac{\gamma_{s}(V)}{2V} (V_{o} - V) \right] + \frac{\gamma_{s}(V)}{V} \left[E(T,V) - E_{oH} \right]$$
 (12)

$$P_{C}(T,V) = \left[\frac{\Gamma(T,V) - \gamma_{s}(V)}{V}\right] \left[E(T,V) - E_{o}(V)\right]$$
(13)

$$E_o = \frac{C^2 - x^2}{2(1 - Sx)} \left[S + \frac{S}{3}x + \frac{S^2}{6} (1 - \frac{\gamma_o}{S})x^2 \right] + E_{oo}(1 + \gamma_o x) + E_{oH}$$
 (14)

$$C_{V} = \frac{3R}{2} \left\{ 1 + \left[\alpha \frac{T}{T_{m}} + 1 \right]^{-1} \right\}, \quad \alpha = .1, T > T_{m}$$
 (15)

$$S_m = S_s + \nu \left[\Delta S - .143R \right] \tag{16}$$

$$A_m = A_s = \nu^2 (\Delta T/2)(\Delta S - .143R)$$
 (17)

$$E_m = E_s + \nu (T - \nu (\Delta T/2))(\Delta S - .143R)$$
 (18)

$$P_{\rm m} = P_{\rm s} + \nu \frac{\lambda T_{\rm m}}{V} (\Delta S - .143R) \tag{19}$$

$$S_1 = S_s - \frac{3R}{2} \ln \left[\frac{\alpha T}{T_m} + 1 \right] + \Delta S$$
 (20)

$$A_1 - A_s - \frac{\Delta T}{2} (\Delta S - .143R) - (T - T_m) \left[\Delta S + \frac{3R}{2} \right]$$

$$+\frac{3RT_{m}}{2\alpha}\left\{\left[\frac{\alpha T}{T_{m}}+1\right]\ln\left[\frac{\alpha T}{T_{m}}+1\right]-(\alpha+1)\ln(\alpha+1)\right\}$$
 (21)

$$E_1 = E_s + T_m \Delta S + .143R \frac{\Delta T}{2}$$

$$+\frac{3RT_{m}}{2\alpha}\left\{\ln\left[\alpha\frac{T}{T_{m}}+1\right]-(\alpha+1)\ln(\alpha+1)-\left[\alpha\frac{T}{T_{m}}-1\right]\right\}$$
 (22)

$$P_1 = P_s - \frac{\lambda T_m}{V} \Delta S$$

$$-\frac{3R\lambda}{2\alpha}\frac{T_{m}}{V}\left\{\ln\left[\alpha\frac{T}{T_{m}}+1\right]-(\alpha+1)\ln(\alpha+1)-\left[\alpha\frac{T}{T_{m}}-1\right]\right\}$$
 (23)

$$P_{v} = \frac{RT}{V} \left[\frac{1 + \eta + \eta^{2} - \eta^{3}}{(1 - \eta)^{3}} \right] - \frac{a}{V^{2}}$$
 (24)

$$E_{v} = \frac{3RT}{2} - \frac{2RT^{2}}{V} \frac{(2-\eta)}{(1-\eta)^{3}} \frac{dV_{b}}{dT} - \frac{a}{V} + E_{s}$$
 (25)

$$A_{v} = \left[\frac{7 - 6\eta + \eta^{2}}{2(1 - \eta)^{3}} - \ln V \right] RT - \frac{3RT}{2} \ln T - \frac{a}{V} + E_{a}$$
 (26)

DISTRIBUTION LIST

No. of Copies Organization

- 12 Commander
 Defense Technical Info Center
 ATTN: DDC-DDA
 Cameron Station
 Alexandria, VA 22314
- 1 Director
 Defense Advanced Research
 Projects Agency
 Tactical Technical Office
 ATTN: Dr. James Tegnelia
 1400 Wilson Boulevard
 Arlington, VA 22209
- 1 Director
 Institute for Defense Analysis
 ATTN: Dr. Bruce J. Whittemore
 400 Army Navy Drive
 Arlington, VA 22202
- 1 Deputy Under Secretary of the Army for Operations Research ATTN: Mr. David Hardison Washington, DC 20310
- 1 Commander
 US Army Materiel Development
 and Readiness Command
 ATTN: DRCDMD-ST
 5001 Eisenhower Avenue
 Alexandria, VA 22333
- 2 Commander
 US Army Armament Research
 and Development Command
 ATTN: DRDAR-TSS
 Dover, NJ 07801
- 1 Commander
 US Army Armament Research
 and Development Command
 ATTN: DRDAR-LCA-L,
 Mr. G. Randers-Pehrson
 Dover, NJ 07801

No. of Copies Organization

- 1 Commander
 US Army ARRADCOM
 Benet Weapons Laboratory
 ATTN: DRDAR-LCB-TL
 Watervliet, NY 12189
- 1 Commander
 US Army Armament Materiel
 Readiness Command
 ATTN: DRSAR-LEP-L, Tech Lib
 Rock Island, IL 61299
- 1 Commander US Army Aviation Research and Development Command ATTN: DRSAV-E P.O. Box 209 St. Louis, MO 63166
- Director
 US Army Air Mobility Research
 and Development Laboratory
 Ames Research Center
 Moffett Field, CA 94035
- 1 Commander US Army Communications Rsch and Development Command ATTN: DRDCO-PPA-SA Fort Monmouth, NJ 07703
- 1 Commander
 US Army Electronics Research
 and Development Command
 Technical Support Activity
 ATTN: DELSD-L
 Fort Monmouth, NJ 07703
- 2 Commander
 US Army Missile Command
 ATTN: DRSMI-R
 DRSMI-YDL
 Redstone Arsenal, AL 35809

DISTRIBUTION LIST

No. of Copies Organization

- 1 Commander
 US Army Tank Automotive
 Research & Development Command
 ATTN: DRDTA-UL
 Warren, MI 48090
- 1 Commander US Army Materials and Mechanics Research Center ATTN: DRXMR-T, Mr. J. Mescall Watertown, MA 02172
- 1 Director
 US Army TRADOC Systems
 Analysis Activity
 ATTN: ATAA-SL, Tech Lib
 White Sands Missile Range,
 NM 88002
- 1 AFATL/DLJW/CPT Raymond Bell Eglin AFB, FL 32542
- 2 Director
 Lawrence Radiation Laboratory
 ATTN: Dr. David Young
 Dr. Richard Grover
 P.O. Box 808
 Livermore, CA 94550
- 1 Science Applications, Inc. ATTN: Mr. Ken Dent 2019 W. Clinton Avenue Suite 800 Huntsville, AL 35805
- 1 University of California
 Los Alamos Scientific Lab
 ATTN: Dr. R. Karpp
 P.O. Box 1663
 Los Alamos, NM 87545

Aberdeen Proving Ground

Dir, USAMSAA
ATTN: DRXSY-D
DRXSY-MP, H. Cohen
Cdr, USATECOM
ATTN: DRSTE-TO-F
Dir, USA CSL, Bldg. E3516
ATTN: DRDAR-CLB-PA

USER EVALUATION OF REPORT

Please take a few minutes to answer the questions below; tear out this sheet and return it to Director, US Army Ballistic Research Laboratory, ARRADCOM, ATTN: DRDAR-TSB, Aberdeen Proving Ground, Maryland 21005. Your comments will provide us with information for improving future reports.

1. BRL Report Number
2. Does this report satisfy a need? (Comment on purpose, related project, or other area of interest for which report will be used.)
3. How, specifically, is the report being used? (Information source, design data or procedure, management procedure, source of ideas, etc.)
4. Has the information in this report led to any quantitative savings as far as man-hours/contract dollars saved, operating costs avoided, efficiencies achieved, etc.? If so, please elaborate.
5. General Comments (Indicate what you think should be changed to make this report and future reports of this type more responsive to your needs, more usable, improve readability, etc.)
6. If you would like to be contacted by the personnel who prepared
this report to raise specific questions or discuss the topic, please fill in the following information.
Name:
Telephone Number:
Organization Address: